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The momentum spectra of nuclear active particles in the cosmic radiation at sea level II. A review of predictions

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Abstract. Computer simulations have been made of the propagation through the atmosphere of protons and pions using a model developed primarily for high energy air shower studies. The results are compared with those from other recent simulations and with measurement. There is little agreement between the results of the various simulations; the results of the present work based upon a simple model and assuming reasonable values for the primary intensity, are in broad agreement with the observational data.

1. Introduction

This paper compares observational data on the unaccompanied nuclear active particles (NAP) in the cosmic radiation at sea level and the results of calculations for the propagation through the atmosphere of the pions and protons. It complements a paper describing a recent measurement of the unaccompanied sea level proton and pion momentum spectra in the near vertical direction—Diggory *et al* (1974)—to be referred to as I. It is also the intention to compare, where possible, the results of the present and earlier calculations in the hadron spectra in an attempt to investigate the consistency of the simulation data.

The sea level NAP spectra depend not only on the propagation of the particles through the atmosphere, but also on the energy spectra of the primary particles. In recent years, several direct measurements of the primary energy spectrum using satellite and balloon-borne equipment have been made. These measurements are apparently not in good agreement either with each other or with the previous estimates of the spectrum obtained by less direct means. A comparison of the measured sea level spectra of NAP with recent model predictions may therefore aid the choice of the correct primary spectrum.

The proton spectrum at sea level is affected by the mean free path for collision with air nuclei and the coefficient of inelasticity in the collision. The pion spectrum on the other hand depends both on corresponding values in pion–air nucleus interactions and upon the details of the multiplicity of produced particles and their energy distribution.

Considerable contemporary interest exists in the detail of high energy nuclear interactions in connection with experiments at large accelerators. Furthermore, our own calculations are based upon models developed specifically for the simulations of cosmic ray extensive air showers (EAS) by primary particles of energy above about 10^{17} eV; the testing of such models in the region of interaction energy between that available from accelerators and in the EAS is thus of some importance.

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2. Previous simulations of the flux of single NAP at sea level

2.1. *General*

During the last 10 years the results of several simulations of the propagation of cosmic rays through the atmosphere have been published (Brooke *et al* 1964a, Pinkau 1964, Pal and Peters 1964, Bull *et al* 1965). These simulations have been based upon various models for the characteristics of high energy nucleon and pion–air nucleus interactions. In this paper a detailed comparison is made of the predictions from several more recent models in an attempt to determine whether consistency can be obtained between the various predictions and whether there is agreement with the measurements of the momentum spectra of the unaccompanied NAP at sea level. The three recent models considered here which differ widely in their assumptions are those of Jabs (1968), O'Brien (1971) and a model developed primarily for EAS studies by J C Earnshaw and K E Turver (1972, unpublished).

The three studies vary considerably in their assumptions for the model for high energy interactions and a summary of the more important features is given in table 1. In addition to using different models for the nuclear interactions, different forms of the spectrum and primary cosmic rays were used in the various studies in deriving the sea level particle fluxes. A satisfactory and a direct comparison of the results is therefore difficult. To overcome this limitation predictions have been made with our own model using several different forms of the primary spectrum in an attempt to allow intercomparison of the predictions by different authors with the same assumed primary spectrum. We may thus obtain some estimate of the sensitivity of the observables to the form of the model for nuclear interactions assumed.

The propagation models here considered are all one-dimensional and are intended to produce predictions for comparison with hadron measurements at or near the zenith. In the calculations the particles are assumed to have propagated through the atmosphere vertically unless specifically stated to the contrary. We consider this assumption to be sufficiently valid for hadrons of the energy to be considered here to make the comparison worthwhile, although we note that it would not be so for the muon component.

2.2. *A brief description of other models*

2.2.1. The model developed by Jabs (1968). The interaction parameters in this model were chosen on the basis of data obtained from experiments using accelerators and cosmic

Table 1. The model for high energy interactions used in the present computer simulations.

Parameter	Present model	Jabs (1968)	O'Brien (1971)
Nucleon interaction length, λ_N(g cm^{-2})	80	70	80
Pion interaction length, λ_π(g cm^{-2})	120	74	80
Mean nucleon–nucleus inelasticity, $\bar{K}_N$	0·5	0·4	0·5
Form of multiplicity law	$E_p^{0\cdot25}$	$\ln(E_p)$	E_p^l where $l = 0{\cdot}216$
Distribution of inelasticity, $g(K)\,dK$	Rectangular between 0·25 and 0·75	$K^2 \exp\left(-\frac{3}{\bar{K}}K\right) dK$	Constant value
Energy spectrum of secondaries	CKP†	CKP†	Power law

† Representation of p-light nuclei interactions suggested by Cocconi *et al* (1961)

rays as sources of high energy particles. The parameters which are listed in table 1 indicate that the values of mean free path for pions and protons are shorter than frequently assumed, that pion interactions are inelastic and that particle multiplicities rise slowly with interaction energy. An important and novel aspect of this model was the assumption that 20% of the secondary particles produced were kaons. The differential equations describing the propagation through the atmosphere of cosmic rays were set up and solutions were obtained by the method of successive generations.

The primary spectrum applicable to this work was that denoted by equation (E) in the summary in § 2.3.3 and is characterized by the energy dependence of the assumed spectral exponent.

2.2.2. The model developed by O'Brien (1971). The main characteristic of energetic interactions in this study was a representation by a power law of the dependence of the multiplicity of secondaries upon the interaction energy, in which the exponent of the power law was selected by O'Brien to be 0·216 in order to fit the available experimental data on the cosmic ray spectrum at sea level. Although good agreement was therefore obtained between the predictions and the then available experimental values, this was perhaps fortuitous since the representation of the energy spectrum of secondaries produced was assumed to be a power law. Kaon production was not considered and all pion interactions were regarded as absorptions, leading to an underestimate of the sea level muon intensities by about 20% according to the data of our present work. The atmospheric cosmic ray fluxes were obtained as analytic solutions to an approximate form of the Boltzmann equation describing the nucleonic cascade in the atmosphere.

The form of the primary spectrum used was not specified but it is considered likely by the present authors that it was that given by Peters (1958) for energies greater than 10 GeV which is

$$\lg N = 6{\cdot}73 - 0{\cdot}0495[11{\cdot}9 + \lg(1{\cdot}7 + E_p)]^2 \text{ cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}$$

for proton primaries, where N is the intensity of protons of kinetic energy greater than E_p (GeV). This spectrum is in good agreement with the spectra used in the present work at energies about 10 GeV per nucleon, but gives significantly more particles at the larger energies. Spectra with the same shape but with different constant terms were used for primary nuclei heavier than protons. None of the primary particle spectra used in other models considered here are directly comparable with this spectra which may be best represented by that given in equation (E) (see § 2.3.3) and which will be used for any comparisons.

2.3. *The present work*

2.3.1. A description of the model. The important parameters required for simulations of the propagation of cosmic rays through the atmosphere using a phenomenological model for the interactions are:

(i) the mean multiplicity of particles produced in pion and nucleon interactions with air nuclei;

(ii) the fraction of the incident energy which is used to produce secondary particles, ie the inelasticity of pion and nucleon interactions with air nuclei;

(iii) the distribution in energy of the particles produced in such interactions;

(iv) the interaction mean free paths of nucleons and pions in air.

The values of the parameters chosen for the present study are discussed below.

The multiplicity of produced particles. A survey of experimental data on nuclear interactions of energies from 19 GeV up to 10 TeV has been made by Hough (1972, private communication) and the parameters used in this model are those resulting from this survey. The mean multiplicity of particles produced in an interaction of a particle with an air nucleus is

$$n_s = 3{\cdot}85(KE_p)^{1/4}$$

where n_s is the number of secondaries, charged and neutral, K is the inelasticity of the interaction and E_p is the kinetic energy of the incident particle in the laboratory frame of reference (in GeV).

The value of the constant arises from the value of 1·9 for pp collisions when allowing for the mass of the target nucleus A using a relation in which the number of secondaries is proportional to $A^{0{\cdot}19}$; it is assumed that all produced particles are pions and that positive, negative and neutral pions are produced in equal numbers. The production of isobars in nucleon interactions has not been allowed for since it appears that the cross section for their production is a decreasing function of energy at accelerator energies up 70 GeV, and, with the exceptions of the interpretations of certain authors (eg Koshiba *et al* 1967), it is considered that the data on the observed distributions of secondary particle energy in high energy interactions may be explained by fluctuations in the interactions.

The coefficient of inelasticity. The distribution of inelasticity in nucleon–nucleus interactions is assumed for most of the calculations to be rectangular between limits of 0·25 and 0·75 leading to a mean value of 0·5. The effects of the form of the distribution in inelasticity have been investigated by employing the distribution of the form considered by Brooke *et al* (1964a) with a similar mean value. The differences in the predicted sea level spectra have been shown to be negligible. We have assumed that pion interactions may be taken to be totally inelastic and we note that this assumption is of little significance as the product of pion interactions does not make a major contribution to the unaccompanied hadron spectrum at sea level.

The energy distribution of the secondary particles. The distribution of the secondary energies in the laboratory frame of reference is taken to follow the formula for proton–light nuclei interactions by Cocconi *et al* (1961) and is of the form

$$S(E_\pi)\,dE_\pi = 0{\cdot}5\left[\frac{1}{G}\exp\left(-\frac{E_\pi}{G}\right)+\frac{1}{T}\exp\left(-\frac{E_\pi}{T}\right)\right]dE_\pi,$$

where G and T are the mean energies in the laboratory frame of those particles moving backward and forward respectively in the centre-of-mass frame of reference.

The interaction mean free path. At energies greater than about 20 GeV the total cross section of the nucleon–nucleon interaction tends to a constant value of 38 mb (Giacomelli 1971) although recent data from accelerators indicate a rising cross section (Amendolia *et al* 1973). Using an optical model for nucleon–nucleus interactions the value of 38 mb corresponds to a nuclear interaction mean free path of 80 g cm^{-2} in air. This mean free path is assumed constant in the range of energy appropriate to the present study. From a consideration of accelerator data for pion interactions the mean free path of pions in air is taken to be 120 g cm^{-2} for both positive and negative pions at all energies.

2.3.2. Computation procedures. In the first instance a simple analytical treatment for the propagation of nucleons through the atmosphere has been made. In this treatment no allowance is made for the known distribution in inelasticity or the fluctuation effects arising from the depths of interactions. Moreover the primary spectrum assumed was that indicated by equation (B) in § 2.3.3 and no cut-off was applied to the spectrum to ensure that the computed flux was that of single, unaccompanied particles. The results of this calculation, shown in figure 1, demonstrate the strong sensitivity of the sea level proton spectrum to the assumed mean values for the mean free path and the coefficient of inelasticity in nucleon–air nucleus interactions. The effects of propagation at different zenith angles are also shown.

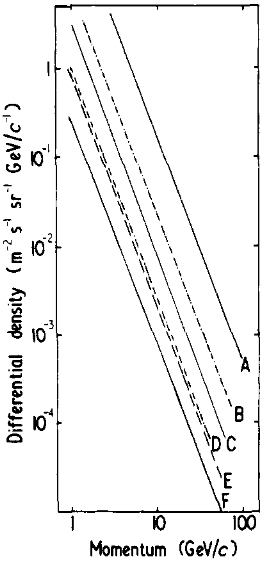

Figure 1. The differential momentum spectrum of protons from an analytic model for the propagation through the atmosphere. The full line represents the results for the model similar to that used in the Monte Carlo simulations. The other lines indicate the effects of changes in zenith angles of propagation, nuclear mean free path and coefficient of inelasticity in nucleon–nucleus interactions. A, $K_N = 0{\cdot}3$; B, $\lambda_N = 90\ \mathrm{g\ cm^{-2}}$; C, normal mode ($\theta = 0°$); D, normal mode ($\theta = 30°$); E, $\lambda_N = 70\ \mathrm{g\ cm^{-2}}$; F, $K_N = 0{\cdot}7$.

Most of the data reported in this paper were obtained from a one-dimensional Monte Carlo simulation of the vertical propagation through the atmosphere of a primary cosmic ray nucleon. The kinetic energy spectra at sea level of nucleons, muons and charged pions have been computed for primary nucleons of energy in the range 10–10^5 GeV. The total number of electrons at sea level has also been predicted using a solution under approximation (B) of the electron cascade equations. Typical data are shown in figure 2 for the number of particles of various types and energies to be observed at sea level for a primary of given energy incident vertically upon the atmosphere.

2.3.3. The primary cosmic ray spectrum. The dependence of the sea level hadron spectra on the form of the primary particle spectrum has already been mentioned. Furthermore the atomic mass number of the primaries is of importance, since it is the energy per primary nucleon which influences the predicted hadron spectra. The possibility of

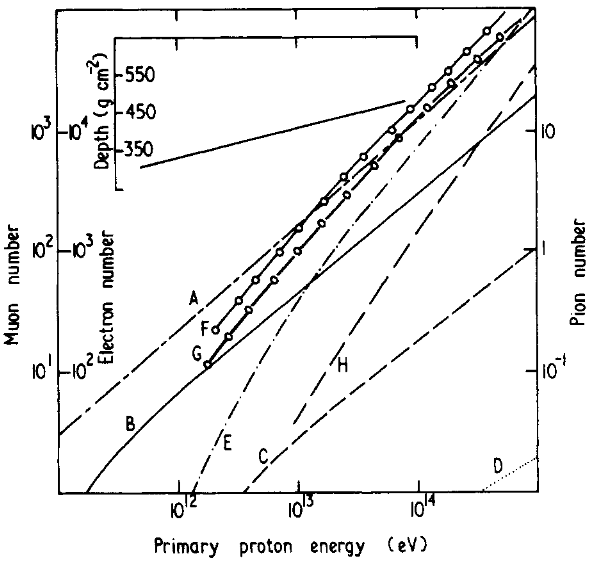

Figure 2. Details of the numbers of muons (A, $N_\mu > 1{\cdot}3$ GeV; B, $N_\mu > 13$ GeV; C, $N_\mu > 133$ GeV; D, $N_\mu > 1333$ GeV), electrons (E) and pions (F, $N_\pi > 1{\cdot}3$ GeV; G, $N_\pi > 13$ GeV; H, $N_\pi > 133$ GeV) produced at sea level by primary protons of energy in the range 10^2–10^6 GeV. Inset are details of the depth of maximum development of the small electron cascades produced by such primary particles.

combining the various measured spectra with our recent simulation data requires a range of primary spectra to be specified. Such a range of spectra expressed in integral form is shown in figure 3. The spectra referring to the total radiation, are with the exception of that of Schmidt *et al* (1969), as quoted by the original authors. In the case of the spectrum of Schmidt *et al*, the total radiation spectrum has been derived by the present authors by combining the spectrum of primary protons with the spectrum of primary α particles given by Webber (1969).

Although the published spectra are frequently quoted as integral spectra, the current work required a differential spectrum (in terms of energy per nucleon) and the following expressions have been derived and used:

$$J(E)\,\mathrm{d}E = 1{\cdot}465\times 10^4 E^{-2{\cdot}6}\,\mathrm{d}E\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1} \qquad \text{(Webber 1967)} \qquad \text{(A)}$$

$$J(E)\,\mathrm{d}E = 1{\cdot}375\times 10^4 E^{-2{\cdot}58}\,\mathrm{d}E\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1} \qquad \text{(Brooke \textit{et al} 1964)} \qquad \text{(B)}$$

$$J(E)\,\mathrm{d}E = 1{\cdot}96\times 10^4 E^{-2{\cdot}7}\,\mathrm{d}E\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1}$$

$$\text{(Schmidt \textit{et al} 1969, Webber 1969)} \qquad \text{(C)}$$

$$J(E)\,\mathrm{d}E = 2{\cdot}54\times 10^4 E^{-2{\cdot}6}\,\mathrm{d}E\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1} \qquad \text{(Grigorov \textit{et al} 1971).} \qquad \text{(D)}$$

In addition to these spectra assumed to have a constant exponent from 10 GeV to 10^5 GeV, the spectrum used by Jabs (1968) in which the exponent is a function of energy of the form shown below has been investigated:

$$J(E)\,\mathrm{d}E = 10^4 E^{-2{\cdot}35(1+0{\cdot}02\,\lg E)}\,\mathrm{d}E\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1}. \qquad \text{(E)}$$

2.3.4. The derivation of sea level spectra. For a series of some 20000 Monte Carlo simulations the average sea level energy spectra for the various types of particle have

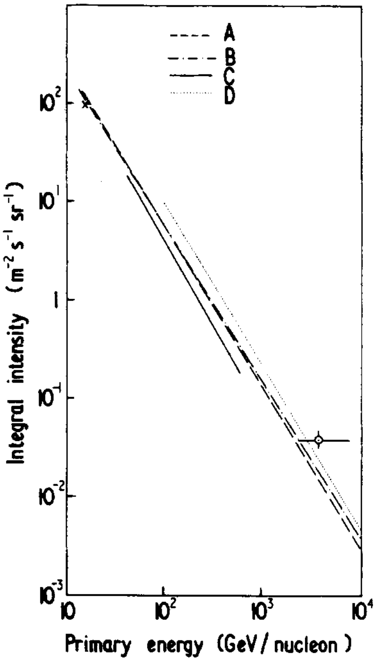

Figure 3. The integral primary energy spectrum of the total radiation. The spectra A–D are described in the text. ⊙, Kaplon and Ritson (1952); × Balasubrahmanian *et al* (1962).

been built up at several primary energies in the range 10–10^5 GeV. The majority of the simulations performed are for low energy primaries since these contribute the greatest weight to the sea level spectra and are susceptible to the largest statistical fluctuations. To derive the intensity of single particles at a given sea level energy the yield from a particular primary energy has been weighted with a primary intensity and the resulting series of values integrated numerically over the appropriate range of primary energy.

An upper limit to this integration has been determined by the probability of detecting an accompanying particle in the instrument used for the measurements. This is an important consideration since the recent and indeed earlier measurements were for single, unaccompanied hadrons. Using the simulations to give shower size and the lateral distribution of the electron components of the extensive air shower generated by a particle of particular primary energy it is considered that there is a significant probability of detecting accompanying particles in the magnetic spectrograph described in I for hadrons derived from a primary particle of energy above 6×10^4 GeV. However, the contribution to the sea level fluxes of the energies considered here for such primaries is small and so the upper limit of integration has been taken for convenience to be 10^5 GeV. We considered such a limit of integration to be appropriate to the earlier measurements. It should be noted that the loss of data in experiments due to accompanying particles has probably not been taken into account by other authors and therefore comparison of predicted spectra at energies above about 50 GeV may be inappropriate. The lower limit of integration is, ideally, the sea level energy under consideration but this has been taken to be 10 GeV, since below this energy the primary spectrum is affected by geomagnetic and solar phenomena and the contribution from primaries of energies less than this is negligible.

3. The predicted proton spectrum at sea level

As has been remarked already, the energy spectrum of primary cosmic rays is often taken to be represented by a power law in energy with an exponent which is either constant over the whole range of energy considered or only a slowly varying function of energy. It has been suggested by Pal and Peters (1964) that the nucleon spectrum will not be changed by diffusion through the atmosphere. This will only be true if the ionization loss of protons and fluctuations in the inelasticity of nucleon–nucleus collisions are ignored and if the mean coefficient of inelasticity is independent of the collision energy. Allowance for these factors, in particular the ionization loss, will change the form of the nucleon spectra in the atmosphere and will lead to a reduction in intensity at momenta of a few GeV/c, where the typical ionization energy loss of about 1 GeV is significant. Furthermore the confinement of the measurement in many cases to recorded single unaccompanied protons reduces the contribution from the more energetic primaries to the flux of energetic protons. We expect therefore that the observed spectrum of single protons will not necessarily follow the shape of the primary spectrum.

The predicted momentum spectra of protons at sea level derived from the models of Jabs and O'Brien and from the present model, with a similar primary spectrum (E) are shown in figure 4. It can be seen that the predictions of Jabs and the present model are in good agreement at momenta less than 10 GeV/c although the flattening of the spectrum at low momentum is not so pronounced in the Jabs prediction, presumably because the ionization energy loss has been neglected. The prediction of O'Brien appears to be significantly lower than that from the present model at all momenta. This discrepancy is difficult to understand since the spectrum of nucleons depends on the values of the interaction length and inelasticity assumed for the nucleon–nucleus interactions used in the model and these should be identical.

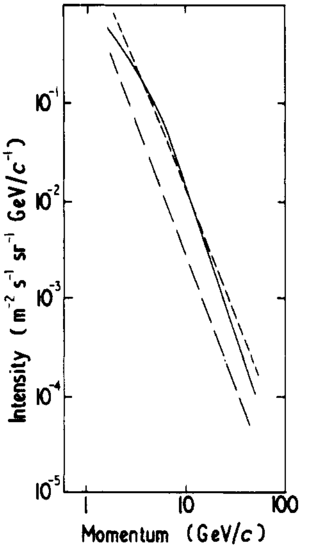

Figure 4. The differential momentum spectrum of single (unaccompanied) protons at sea level from various simulations. Full curve, present work; broken curve, Jabs (1968); chain curve, O'Brien (1971).

The proton spectrum at ground level as reported in I is shown in figure 5 and there is seen to be reasonable agreement between the measurements and the predictions of our present study assuming any of the primary spectra except (D). Here the data refer to a zenith angle range of 0–20 deg; calculations have been made for an angle of 10 deg. It can be seen that the spectrum at low momenta up to 20 GeV/*c* is insensitive to the form of the primary spectrum. The explanation may be seen in figure 6 where a curve showing the contributions of particles at various primary energies to the sea level flux of vertical protons at momentum 5·5 GeV/*c* and 100 GeV/*c* are shown.

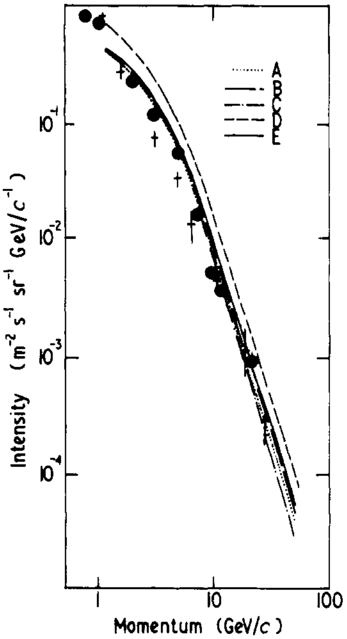

Figure 5. The differential momentum spectrum of unaccompanied protons at sea level. ●, present experiment; +, Brooke and Wolfendale (1964). The curves correspond to the spectra derived from our recent model simulations when the various primary spectra described in the text are assumed and protons are assumed incident at a zenith angle of 10 deg.

4. The predicted spectrum of negative pions

The momentum spectrum of negative pions predicted using the various models and primary spectra described above are shown in figure 7. The three models do not produce consistent predictions over the whole range of momentum although the results from the present model are in good agreement with those of Jabs at low momenta. The most important apparent difference between the model of Jabs and that described here is in the law relating to the mean multiplicity of particles produced in a nuclear interaction to the kinetic energy of the incident particle. Jabs used an exponential dependence on energy in contrast to the power law with exponent 0·25 of the present models. The mean multiplicity produced by these two expressions are however almost identical for interaction energies which are important for the pions of low momenta. At the higher interaction energies this is not so and the discrepancy between the two laws increases with

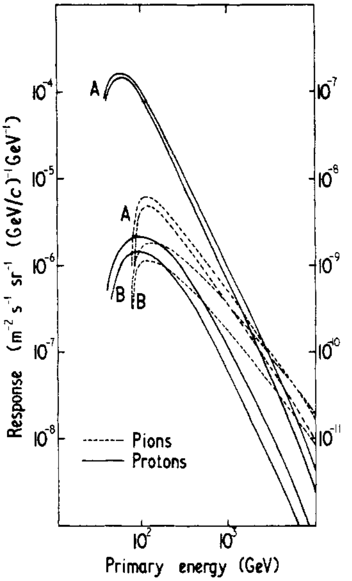

Figure 6. The contributions of primary protons of energy in the range 10–10^4 GeV to the flux of 5·5 GeV/*c* (curves A, left-hand axis) and 100 GeV/*c* (curves B, right-hand axis) pions and protons at sea level.

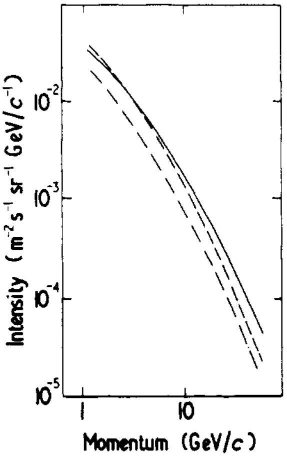

Figure 7. The differential momentum spectrum of unaccompanied negative pions at sea level from various simulations. Full curve, present work; broken curve, Jabs (1968); chain curve, O'Brien (1971).

increasing energy so that overall agreement should not be expected between the predictions for the two models.

The multiplicity laws used by O'Brien and the present model are rather similar and so it is not unreasonable that the spectral shapes should be the same. It is noted that the comparison of the predicted proton spectra from the two models also show a large discrepancy in intensity which may have a common origin with the discrepancy between the predicted pion spectra. The omission of a consideration of the pions produced in pion–nucleus interactions by O'Brien must also cause an underestimate of the pion flux.

Figure 8 shows the experimental data obtained in I and those reported by Brooke *et al* (1964b) together with the predictions from the present model for a zenith angle of 10 deg. It can be seen that the intensity of the pion component at the momenta measured in the present experiment is more sensitive to the primary spectrum than the spectrum of protons. However because of the low flux of pions compared with that of muons and protons, an accurate measurement is difficult for pions and full advantage has not yet been taken of this greater sensitivity.

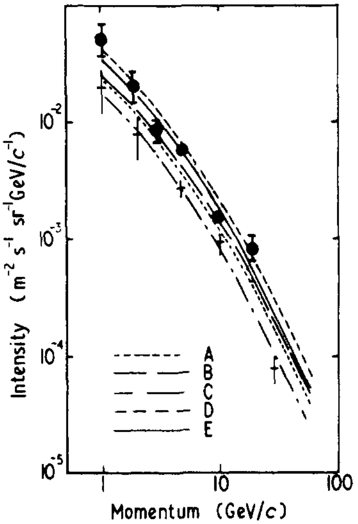

Figure 8. The differential momentum spectrum of unaccompanied negative pions at sea level. ●, present experiment; +, Brooke *et al* (1964b). The curves correspond to the spectra derived from our recent model simulations when the various primary spectra described in the text are assumed and the pions are incident at a zenith angle of 10 deg.

5. The pion/proton ratio

The quantity measured directly in the experiment discussed in I and presumably in the earlier measurements is the ratio of the flux of single unaccompanied negative pions to the flux of positive pions and protons, in the assumed absence of other types of nuclear active particles. Such measurements are therefore specific to detectors of given dimensions. It is interesting, therefore, to compare the measured value of the ratio of the number of single pions to protons measured in the equipment of I, derived assuming that there are no antiprotons at sea level, with that predicted from our models using different values for the primary spectrum. This is done in figure 9 and it is seen that there is an apparent limited sensitivity of the pion/proton ratio, arising from the sensitivity of the intensity of the pion component to the form of the primary spectrum. Those primary spectra considered here all appear to represent the shape of the pion/proton ratio as a function of momentum. The observed minimum in the ratio at about 3 GeV/*c* appears to be predicted as does the increase to the limit of the momentum investigated in I. Such an increase is a consequence of the specification of our measurement—single particles incident on the instrument described in I—which results in a decrease in the

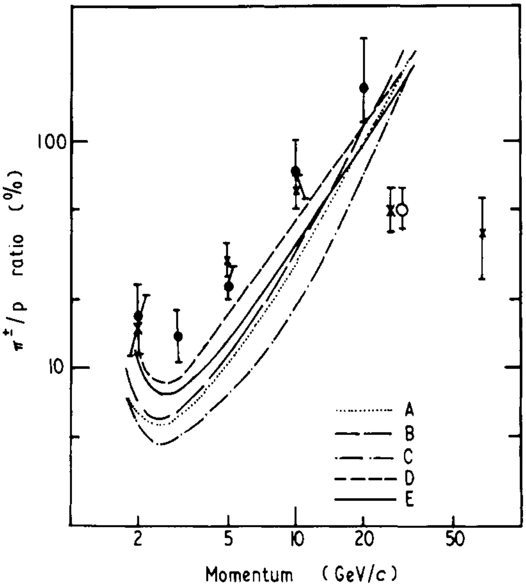

Figure 9. The $\pi^{\pm}/p$ ratio for single particles at sea level observed in the instrument discussed in I. ɸ, present experiment; ⫳ Brooke *et al* (1964b); ɸ, Subrahmanian (1962). The model predictions are also specific to that instrument and relate to particles recorded at a zenith angle of 10 deg.

proton intensities at momenta of a few GeV/*c* (see figure 4). This decrease combines with the smaller number of pions arising as a consequence of the increasing decay probability to give the predicted overall increase in the ratio.

We note that a decrease as large as that in the earlier measurement of the $\pi^{\pm}/p$ ratio at high momenta would not be expected on the basis of the present calculations, even if all detected particles were unaccompanied.

Overall however, the absolute value of the $\pi^{\pm}/p$ ratio predicted from our simulations is lower than that observed. This discrepancy may readily be removed by minor adjustments to the assumed values of a nucleon mean free path and/or the inelasticity. (We note that the pion intensities may not be so readily varied.) It is however a prerequisite of any such adjustment of the model that the cascade propagation in a non-vertical direction be adequately considered. (An indication of the strong sensitivity of a proton spectrum to the variations in the nucleon mean free path, coefficient of inelasticity and, in particular, the zenith angle considered is shown by the results from the simple non-fluctuating treatment of the proton propagation given in figure 1.) The measurement reported in I was made at zenith angles in the range 0–20 deg with a typical value of about 10 deg. We consider that the present measurement data and simulation results are in satisfactory agreement and do not warrant changes in the model assumed.

6. Conclusions

The detailed comparison of recent predicted hadron fluxes at sea level is difficult due to the variations both in the assumed primary particle spectrum and the detail of the interaction models. However even in those cases where the variables are similar, or sufficiently so to make comparison worthwhile, there are apparently discrepancies between the resultant predictions of the various authors.

A comparison of our own calculated proton and pion spectra with those measured experimentally in the range of momentum 1–30 GeV/*c* indicate that:

(i) In this momentum range the sea level proton spectrum is dependent upon the primary spectrum in the range 10–1000 GeV. Equally good representations of the measured spectra are given by calculations based on any of the recent measured primary spectra which show only minor differences in this energy region.

(ii) The choice of the coefficient of inelasticity in a nucleon–air nucleus interaction in the energy range 10–1000 GeV as 0·5 and the mean free path for nucleon–air nucleus interactions as $80\,\mathrm{g\,cm^{-2}}$ produce data in acceptable agreement with measurement. We do not consider that the accuracy of either measurement or simulation data at present warrant changes in these values which are based upon direct measurement at lower energies.

(iii) The pion spectrum is sensitive to the primary spectrum in the range 100–10 000 GeV but advantage cannot be taken at present of this limited sensitivity due to the statistical uncertainties in the measured pion spectrum.

(iv) The shape of the observed pion/proton ratio in the range 1–30 GeV/*c* is well accounted for by the predictions from our air shower model. The predicted absolute values for the ratio are somewhat lower than the observed ratio. Again, we consider that this discrepancy may be resolved by refinements of either the measurement or simulation procedures.

We conclude that although our simple air shower model lacks many of the refinements suggested by recent experiments at accelerators, it accounts adequately for the hadron spectra in the cosmic ray beam at sea level.

Acknowledgments

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References

Amendolia S R *et al* 1973 *Phys. Lett.* **44B** 119–24
Balasubrahmanian V K *et al* 1962 *J. Phys. Soc. Japan* **17** *Suppl.* AIII 8–12
Brooke G and Wolfendale A W 1964 *Proc. Phys. Soc.* **83** 843–52
Brooke G *et al* 1964a *Proc. Phys. Soc.* **83** 853–69
—— 1964b *Proc. Phys. Soc.* **83** 871–7
Bull R M *et al* 1965 *Nuovo Cim.* A **40** 365–84
Cocconi G, Koester L G and Perkins D H 1961 UCRL-10022, 167–90
Diggory I S *et al* 1974 *J. Phys.* A: *Math., Nucl. Gen.* **7** 741–64
Giacomelli G 1971 *Progress in Nuclear Physics* vol 12 (Oxford: Pergamon) pp 77–163
Grigorov N L *et al* 1971 *Proc. 12th Int. Conf. on Cosmic Rays, Hobart* vol 5 (Hobart: University of Tasmania) pp 1746–51
Kaplon M F and Ritson D M 1952 *Phys. Rev.* **88** 386–97
Koshiba M *et al* 1967 *Can. J. Phys.* **46** 671–4

Jabs A 1968 *Z. Phys.* **212** 222–34

O'Brien K 1971 *Nuovo Cim.* A 3 521–47

Pal Y and Peters B 1964 *K. danske Vidensk. Selsk., Math.-fys. Meddr* **33** 15–55

Peters B 1958 *Handbook of Physics* ed E U Condon and H Odishaw (New York: McGraw-Hill) pp 9/204–45

Pinkau K 1964 *Fortschr. Phys.* **12** 139–234

Schmidt W K H *et al* 1969 *Phys. Rev.* **184** 1279–82

Subrahmanian A 1962 *PhD Thesis* University of Madras

Webber W R 1967 *Handb. Phys.* **46** 181–264 (Berlin: Springer-Verlag)

—— 1969 *Proc. 11th Int. Conf. on Cosmic Rays, Budapest* (Budapest: Central Research Institute for Physics) pp 275–314